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This Volume of

INSPIRE

is being dedicated to

BRAHMAGUPTA: MATHEMATICIAN AND ASTRONOMER

Brahmagupta (born c. 598CE, died c. 668 CE) was an Indian mathematician and astronomer. The great 7th Century Indian mathematician and astronomer Brahmagupta wrote some important works on both mathematics and astronomy. He was from the state of Rajasthan of northwest India (he is often referred to as Bhillamalacarya, the teacher from Bhillamala), and later became the head of the astronomical observatory at Ujjain in central India. Most of his works are composed in elliptic verse, a common practice in Indian mathematics at the time, and consequently have something of a poetic ring to them.

He is the author of two early works on mathematics and astronomy: the *Brāhmasphuṭasiddhānta* a theoretical treatise, and the *Khaṇḍakhādyaka* a more practical text. Brahmagupta was the first to give rules to compute with *zero*. The texts composed by Brahmagupta were in elliptic verse in Sanskrit, as was common practice in Indian mathematics.

FOREWORD

The present volume of ***INSPIRE*** contains the various research papers of Faculty and Research Scholars of Department of Mathematics, INSTITUTE FOR EXCELLENCE IN HIGHER EDUCATION, BHOPAL (M. P.).

For me it is the realization of a dream which some of us have been nurturing for long and has now taken a concrete shape through the frantic efforts and good wishes of our dedicated band of research workers in our country, in the important area of mathematics.

The editor deserves to be congratulated for this very successful venture. The subject matter has been nicely and systematically presented and is expected to be of use to the workers.

(Dr. Geeta Saxena)
Director & Patron
IEHE, Bhopal (M. P.)

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A COMMON FIXED POINT THEOREM IN CONE METRIC SPACES THROUGH RATIONAL EXPRESSIONS

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ABSTRACT: In this paper we have proved a common fixed point theorem in cone metric space for rational inequality in normal cone setting. Our result generalize several fixed point results in cone metric spaces and in ordinary metric spaces as special cases.

KEYWORDS: Cone metric spaces, common fixed Point, rational inequality.

MATHEMATICS SUBJECT CLASSIFICATION: Primary 47H10, Secondary 54H25

1. INTRODUCTION AND PRELIMINARIES:

Huang and Zhang [5] initiated cone metric spaces, which is a generalization of ordinary metric spaces, by substituting the real numbers with ordered Banach spaces. Consistent with Huang and Zhang [5], the following definitions and results will be needed in the sequel.

Definition 1.1([5]): Let E be a real Banach space.A subset P of E is called a cone if and only if:

(a) P is closed, non-empty and $P \neq \{0\}$,
(b) $a, b \in R$, $a, b \geq 0$, $x,y \in P$ implies that $ax + by = P$,
(c) $P \cap (-P) = \{0\}$.

Given a cone $P \subset E$, we define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. A cone P is called normal if there is a $K > 0$ such that for all $x, y \in E$, $0 \leq \mathbf{x} \leq \mathbf{y}$ implies

$$(\mathbf{1.1.1}) \quad \|\boldsymbol{x}\| \leq K\|\boldsymbol{y}\|$$

The least positive number satisfying the above inequality is called the normal constant of P, while $x << y$ but $x \neq y$.

Definition 1.2 ([5]): Let X be a non-empty set. Suppose that the mapping $d : X\times X \to E$ satisfies :

(d_1) $0 \leq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$,
(d_2) $d(x, y) = d(y, x)$ for all $x, y \in X$,
(d_3) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a cone metric on X and (X, d) is called a cone metric spaces. The concept of cone metric space is more general than that of a metric space.

Definition 1.3 ([5]) : Let (X, d) be a cone metric space, $\{x_n\}$ be a sequence in X and $x \in X$. Then for every $c \in E$ with $0 << c$, we say that:

(i) $\{x_n\}$ is a convergent sequence if there is a positive integer N such that, for all n > N,

$d(x_n, x) << c$ for some x ∈ X.

(ii) $\{x_n\}$ is a Cauchy sequence if there is a positive integer N such that, for all n, m > N,

$d(x_n, x_m) << c$.

A cone metric space X is said to be complete if every Cauchy sequence in X is convergent in x. It is known that a sequence $\{x_n\}$ converges to x ∈ X if and only if

$d(\{x_n, x) \to 0$ as $n \to \infty$.

The limit of a convergent sequence is unique provided P is a normal cone with normal constant K (see, [4], [5]).

2. MAIN RESULTS:

Theorem 2.1 : Let (X, d) be a complete cone metric space, and P a normal cone with normal constant K. Let f and g are two self mappings of X satisfying

(2.1.1) $$d(fx, gy) \le \alpha\, d(x, y) + \beta \frac{d(x,fx)d(y,gy)}{1+d(x,y)} + \gamma \frac{d(y,fx)d(x,gy)}{1+d(x,y)} + \delta \frac{d(x,fx)d(y,gy)}{1+d(x,y)} + \eta \frac{d(y,fx)d(y,gy)}{1+d(x,y)}$$

For all x, y ∈ X, where α, β, γ, δ, η ≥ 0 and α + β + γ +δ + η < 1**.** Then f and g have a unique common fixed point in X.

Proof: Let $x_0 \in X$ be arbitrary. We define a sequence $\{x_n\}$ by $x_{2n+1} = fx_{2n}$, $x_{2n+2} = gx_{2n+1}$, n = 0, 1, 2, . . .

Now,

$$\begin{aligned} d(x_{2n+1}, x_{2n+2}) &= d(fx_{2n}, gx_{2n+1}) \\ &\le \alpha\, d(x_{2n}, x_{2n+1}) + \beta \frac{d(x_{2n},fx_{2n})d(x_{2n+1},gx_{2n+1})}{1+d(x_{2n},x_{2n+1})} \\ &\quad + \gamma \frac{d(x_{2n+1},fx_{2n})d(x_{2n},gx_{2n+1})}{1+d(x_{2n},x_{2n+1})} \\ &\quad + \delta \frac{d(x_{2n},fx_{2n})d(x_{2n+1},gx_{2n+1})}{1+d(x_{2n},x_{2n+1})} + \eta \frac{d(x_{2n+1},fx_{2n})d(x_{2n+1},gx_{2n+1})}{1+d(x_{2n},x_{2n+1})} \\ &= \alpha\, d(x_{2n}, x_{2n+1}) + \beta \frac{d(x_{2n},x_{2n+1})d(x_{2n+1},x_{2n+2})}{1+d(x_{2n},x_{2n+1})} \\ &\quad + \gamma \frac{d(x_{2n+1},x_{2n+1})d(x_{2n},x_{2n+2})}{1+d(x_{2n},x_{2n+1})} + \\ &\quad \delta \frac{d(x_{2n},x_{2n+1})d(x_{2n+1},x_{2n+2})}{1+d(x_{2n},x_{2n+1})} + \eta \frac{d(x_{2n+1},x_{2n+1})d(x_{2n+1},x_{2n+2})}{1+d(x_{2n},x_{2n+1})} \\ &< \alpha d(x_{2n}, x_{2n+1}) + \beta d(x_{2n}, x_{2n+1}) + \delta d(x_{2n}, x_{2n+1}) \end{aligned}$$

By (1.2)(d_1) and using the fact that $1 + d(x_{2n}, x_{2n+1}) > d(x_{2n}, x_{2n+1})$

Which implies that

(2.1.2) $d(x_{2n+1}, x_{2n+2}) \le q d(x_{2n+1}, x_{2n+2})$

Where, $q = \frac{\alpha}{1-\beta+\delta} < 1$.

Similarly, it can be shown that

$$d(x_{2n+3}, x_{2n+2}) \leq qd(x_{2n+2}, x_{2n+1})$$

Therefore, for all n,

$$\begin{aligned} d(x_{n+1}, x_{n+2}) &\leq qd(x_n, x_{n+1}) \\ &\leq q^2 d(x_{n-1}, x_n) \\ &\leq \dots \dots \dots \\ &\leq q^{n+1} d(x_0, x_1) \end{aligned}$$

Now, for any m > n, we have

$$\begin{aligned} d(x_n, x_m) &\leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \ . \ . \ . \ + d(x_{m-1}, x_m) \\ &\leq (q^n + q^{n+1} + q^{n+2} + \ . \ . \ . + q^{m-1}) d(x_0, x_1) \\ &\leq \frac{q^n}{1-q} d(x_0, x_1) \end{aligned}$$

From (1.1.1)

$$\|d(x_n, x_m)\| \leq \frac{q^n}{1-q} K \|d(x_0, x_1)\|$$

Which implies that $d(x_n, x_m) \to 0$ as n, m $\to \infty$.

Hence $\{x_n\}$ be a Cauchy sequence.

Since, X is complete, there exists a point u in X such that $x_n \to$ u as n $\to \infty$. Now, we have from (2.1.1)

$$\begin{aligned} d(u, gu) &\leq d(u, x_{2n+1}) + d(x_{2n+1}, gu) \\ &= d(u, x_{2n+1}) + d(fx_{2n}, gu) \\ &\leq d(u, x_{2n+1}) + \alpha\, d(x_{2n}, u) + \beta \frac{d(x_{2n}, fx_{2n}) d(u, gu)}{1+d(x_{2n}, u)} + \gamma \frac{d(u, fx_{2n}) d(x_{2n}, gu)}{1+d(x_{2n}, u)} \\ &\quad + \delta \frac{d(x_{2n}, fx_{2n}) d(u, gu)}{1+d(x_{2n}, u)} + \eta \frac{d(u, fx_{2n}) d(u, gu)}{1+d(x_{2n}, u)} \\ &= d(u, x_{2n+1}) + \alpha\, d(x_{2n}, u) + \beta \frac{d(x_{2n}, x_{2n+1}) d(u, gu)}{1+d(x_{2n}, u)} + \gamma \frac{d(u, x_{2n+1}) d(x_{2n}, gu)}{1+d(x_{2n}, u)} \\ &\quad + \delta \frac{d(x_{2n}, x_{2n+1}) d(u, gu)}{1+d(x_{2n}, u)} + \eta \frac{d(u, x_{2n+1}) d(u, gu)}{1+d(x_{2n}, u)} \end{aligned}$$

From (1.1.1) this implies that

$$\begin{aligned} \|d(u, gu)\| \leq K \| & d(u, x_{2n+1}) + \alpha\, d(x_{2n}, u) + \beta \frac{d(x_{2n}, x_{2n+1}) d(u, gu)}{1+d(x_{2n}, u)} \\ & + \gamma \frac{d(u, x_{2n+1}) d(x_{2n}, gu)}{1+d(x_{2n}, u)} + \delta \frac{d(x_{2n}, x_{2n+1}) d(u, gu)}{1+d(x_{2n}, u)} + \eta \frac{d(u, x_{2n+1}) d(u, gu)}{1+d(x_{2n}, u)} \| \end{aligned}$$

Now, right hand side of the above inequality approaches to 0 as n $\to \infty$.

Hence, $\|d(u, gu)\| = 0$ and so that u = gu.

Now, again from (2.1.1), we have

$$\begin{aligned} d(fu, u) &= d(fu, gu) \\ &\leq \alpha\, d(u, u) + \beta \frac{d(u, fu) d(u, gu)}{1+d(u, u)} + \gamma \frac{d(u, fu) d(u, gu)}{1+d(u, u)} \\ &\quad + \delta \frac{d(u, fu) d(u, gu)}{1+d(u, u)} + \eta \frac{d(u, fu) d(u, gu)}{1+d(u, u)} \end{aligned}$$

Which gives, by using the definition of partial ordering on E and the properties of cone P,

d(fu, u) = 0 and hence fu = u.

Thus, u is a common fixed point of f and g in X.

To prove uniqueness, let v be an another common fixed point of f and g in X, then

$$d(u, v) = d(fu, gv)$$
$$\leq \alpha\, d(u, v) + \beta \frac{d(u,fu)d(v,gv)}{1+d(u,v)} + \gamma \frac{d(v,fu)d(u,gv)}{1+d(u,v)} + \delta \frac{d(u,fu)d(v,gv)}{1+d(u,v)} + \eta \frac{d(v,fu)d(v,gv)}{1+d(u,v)}$$
$$= (\alpha + \gamma)\, d(u, v)$$

By (1.2)(d_1) and using the fact that $1 + d(x_{2n}, x_{2n+1}) > d(x_{2n}, x_{2n+1})$

Which implies that, d(u, v) = 0 and u = v.

This completes the proof of the theorem.

Corollary 2.2 : Let (X, d) be a complete cone metric space, and P a normal cone with normal constant K. Let f be a self mappings of X satisfying

(2.2.1) $$d(f^p x, f^q y) \leq \alpha\, d(x, y) + \beta \frac{d(x,f^p x)d(y,f^q y)}{1+d(x,y)} + \gamma \frac{d(y,f^p x)d(x,f^q y)}{1+d(x,y)} + \delta \frac{d(x,f^p x)d(y,f^q y)}{1+d(x,y)} + \eta \frac{d(y,f^p x)d(y,f^q y)}{1+d(x,y)}$$

For all x, y ∈ X, where $\alpha, \beta, \gamma, \delta, \eta \geq 0$ and $\alpha + \beta + \gamma + \delta + \eta < 1$. Then f has a unique fixed point in X.

Proof : Inequality (2.2.1) is obtained from (2.1.1) by setting $f \equiv f^p$ and $g \equiv f^q$.

The result follows from theorem 2.1.

Corollary 2.3 : Let (X, d) be a complete cone metric space, and P a normal cone with normal constant K. Let f be a self mappings of X satisfying

(2.3.1) $$d(fx, fy) \leq \alpha\, d(x, y) + \beta \frac{d(x,fx)d(y,fy)}{1+d(x,y)} + \gamma \frac{d(y,fx)d(x,fy)}{1+d(x,y)} + \delta \frac{d(y,fx)d(y,fy)}{1+d(x,y)}$$

For all x, y ∈ X, where $\alpha, \beta, \gamma, \delta \geq 0$ and $\alpha + \beta + \gamma + \delta < 1$. Then f has a unique fixed point in X.

Proof : Set, p = q = 1 in Corollary 2.2.

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APPLICATION OF H-FUNCTION IN PHYSICAL CHEMISTRY

by

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ABSTRACT

The aim of this paper is to represent the equation of Lambert's law by Fox's H–Function of one variable.

1. INTRODUCTION:

The H-function of one variable [2, p.10] is defined as:

$$H_{p,q}^{m,n}\left[x \,\middle|\, \begin{matrix}(a_j, \alpha_j)_{1,p} \\ (b_j, \beta_j)_{1,q}\end{matrix}\right] = (1/2\pi i) \int_L \theta(s)\, x^s\, ds \qquad (1.1)$$

where $i = \sqrt{(-1)}$,

$$\theta(s) = \frac{\prod_{j=1}^{m} \Gamma(b_j - \beta_j s) \prod_{j=1}^{n} \Gamma(1 - a_j + \alpha_j s)}{\prod_{j=m+1}^{q} \Gamma(1 - b_j + \beta_j s) \prod_{j=n+1}^{p} \Gamma(a_j - \alpha_j s)}$$

where

$$\sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j \equiv M > 0, \qquad (1.2)$$

and $|\arg x| < \frac{1}{2} M\pi$.

Let I be the intensity of incident light of wave length l, t be the thickness of medium, then Lambert's law mathematically [1] denoted as

$$dI/dt = -kI$$

or

$$dI/I = -k\, dt \qquad (1.3)$$

2. MAIN RESULT:

The equation of Lambert's law in terms of H-function of one variable to be represented is:

$$\int H^{m+1,\, n}_{p+1,\, q+1}\left[x \,\middle|\, \begin{matrix} (a_j, \alpha_j)_{1,\, p}, (1+I, I_1) \\ (I, I_1), (b_j, \beta_j)_{1,\, q} \end{matrix}\right] dI$$

$$= -\, k\, H^{m,\, n+1}_{p+1,\, q+1}\left[x \,\middle|\, \begin{matrix} (-t, t_1), (a_j, \alpha_j)_{1,\, p}, \\ (b_j, \beta_j)_{1,\, q}, (1-t, t_1) \end{matrix}\right] + c\, H^{m,\, n}_{p,\, q}\, [x], \qquad (2.1)$$

valid for $I > I_1$, $t > t_1$ and $|\arg x| < ½\, \pi M$, where M is given in (1.2).

3. PROOF OF THE FORMULA:

On integrating, (1.3) provides

$$\int dI/I = -\, k \int dt + c$$

or $$\int [\Gamma(I)/\, \Gamma(I+1)] = -\, kt + c$$

or $$\int [\Gamma(I)/\, \Gamma(I+1)] = -\, k\, [\Gamma(t+1)/\, \Gamma(t)] + c \qquad (3.1)$$

where c is integral constant.

Again put $t = t + t_1 s$, $I = I - I_1 s$ (since as thickness of medium increases, intensity of light decrease) in (3.1) and multiply both side by $(1/2\pi i)\theta(s)x^s$, further integrate with respect to s in the direction of contour L and use (1.1), we get (2.1).

4. SPECIAL CASES:

On specializing the parameters, H-function may be reduced to G-function, Lauricella's functions Legendre functions, Bessel functions, hypergeometric functions, Appell's functions, Kampe de Feriet's functions and several other higher transcendental functions. Therefore the result (2.1) is of general nature and may reduced to be in different forms, which will be useful in the literature on applied Mathematics and other branches.

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A SIMPLE MATHEMATICAL MODEL FOR FISHERIES

By

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ABSTRACT

The aim of this paper is to present simple mathematical model for fisheries.

1. INTRODUCTION:

In order to get a first impression of the kind of problems arising in fisheries, let us examine the following situation. A population of a certain fish species inhabits a part of the sea. The population proliferates, but not indefinitely; the sea is only able to support a certain amount of fish and therefore has a finite carrying capacity, which, for this species, we shall assume to have the value K. It seems reasonable to assume that the population will grow logistically, i.e. according to the equation

$$x'(t) = rx(t)\,\{1 - x(t)/k\} \qquad (1)$$

The population is commercially exploited by fishermen, and the equation must therefore contain an extra term accounting for the fishing activity. What will this term be like? In particular: will it depend on the population size x?

2. SIMPLE MATHEMATICAL MODEL:

One can think of two simple mechanisms: a 'blind' way of fishing, and a 'purposeful' way. In the first, the fisherman has no way of locating the fish, and simply drags his net through the water. In that case, his catch can he assumed to be promotional to the amount of fish present:

$$x'(t) = rx(t)\,\{1 - x(t)/k\} - Ex(t) \qquad (2)$$

where E is a proportionality constant (with dimension Γ^{1}) which is built up from such quantities as size of the net, speed of the ship, time spent fishing, size of the sea etc. E is usually called the effort (though the term is also used in various other interpretations).

In the second case the fisherman knows precisely where the fish is, and is able to decide how many fish he will catch in a given amount of time. The model now assumes the form

$$x'(t) = rx(t)\{1 - x(t)/k\} - H \quad (3)$$

where H is the harvesting rate (expressed in amount of fish per time) as determined by the fisherman.

3. CONCLUSION:

For both cases, we can examine the effect of fishing on population growth, the optimalization problem (how to maximize the catch), and the stability problem (the risk of exterminating the population).

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DETERMINATION OF THE CYCLIC PROCESS BY FOX'S H-FUNCTION

By

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ABSTRACT

The aim of this paper is to determine the equation of cyclic process by Fox's H–Function of one variable.

1. INTRODUCTION:

The H-function of one variable [2, p.10] is defined as:

$$H^{m,\,n}_{p,\,q}\left[x \,\middle|\, \begin{matrix}(a_j, \alpha_j)_{1,\,p}\\(b_j, \beta_j)_{1,\,q}\end{matrix}\right] = (1/2\pi i)\int_L \theta(s)\, x^s\, ds \qquad (1.1)$$

where $i = \sqrt{(-1)}$,

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\ \prod_{j=1}^{n}\Gamma(1 - a_j + \alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1 - b_j + \beta_j s)\ \prod_{j=n+1}^{p}\Gamma(a_j - \alpha_j s)}$$

where

$$\sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p}\alpha_j + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q}\beta_j \equiv M > 0, \qquad (1.2)$$

and $|\arg x| < \frac{1}{2} M\pi$.

When a system undergoes a series of physical changes and then returns to its original position then the change in its internal energy is zero because initial and final positions are same, i.e. dE = 0 The amount of heat dQ subjected to system is governed by the first law of thermodynamics

$$dQ = dW \qquad (1.3)$$

Such a process is known as cyclic process [1, p.273].

2. MAIN RESULT:

The equation of cyclic process in terms of H-function of one variable to be represented is:

$$H^{m+1,\,n}_{p+1,\,q+1}\left[x \,\middle|\, \begin{matrix}(a_j, \alpha_j)_{1,\,p},\ (1 + Q, Q_1)\\(Q, Q_1),\ (b_j, \beta_j)_{1,\,q}\end{matrix}\right]$$

$$= \quad H^{m,\, n+1}_{p+1,\, q+1}\left[x \,\middle|\, \begin{matrix} (-W, W_1), (a_j, \alpha_j)_{1,\, p}, \\ (b_j, \beta_j)_{1,\, q}, (1 - W, W_1) \end{matrix}\right] + c\, H^{m,\, n}_{p,\, q}\, [x], \qquad (2.1)$$

valid for $y > y_1$, $t > t_1$ and $|\arg x| < \frac{1}{2}\pi M$, where M is given in (1.2).

3. PROOF OF THE FORMULA:

On integrating, (1.3) provides

$$\int dQ = \int dW + c$$

or $\quad Q = W + c$

or $\quad \Gamma(Q + 1)/\, \Gamma(Q) = \Gamma(W + 1)/\, \Gamma(W) + c \qquad (3.1)$

where c is constant.

Again put $W = W + W_1 s$, $Q = Q - Q_1 s$ (since as work increase, the amount of heat will be decreases) in (3.1) and multiply both side by $(1/2\pi i)\theta(s)x^s$, further integrate with respect to s in the direction of contour L and use (1.1), we get (2.1).

4. SPECIAL CASES:

On specializing the parameters, H-function may be reduced to G-function, Lauricella's functions Legendre functions, Bessel functions, hypergeometric functions, Appell's functions, Kampe de Feriet's functions and several other higher transcendental functions. Therefore the result (2.1) is of general nature and may reduced to be in different forms, which will be useful in the literature on applied Mathematics and other branches.

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GENERALIZED HYPERGEOMETRIC FUNCTIONS AND AN ATOMIC WASTE DISPOSAL PROBLEM

by

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ABSTRACT

The aim of this paper is to determine the solution of a mathematical equation related to 'Atomic Waste Disposal Problem' with the help of generalized hypergeometric function.

1. INTRODUCTION:

For several years the Atomic Energy Commission (now known as the Nuclear Regulatory Commission) had disposed of concentrated radioactive waste material by placing it in tightly sealed drums, which were then dumped at sea in fifty fathoms (300 feet) of water. When concerned ecologists and scientists questioned this practice, they were assured by the A.E.C. that the drums would never develop leaks. Exhaustive tests on the drums proved the A.E.C. right. However, several engineers then raised the question of whether the drums could crack from the Impact of hitting the ocean floor. "Never," said the A.E.C. "We'll see about that," said the engineers. After performing numerous experiments, the engineers found that the drums could crack on impact if their velocity exceeded forty feet per second. The problem before us, therefore, is to compute the velocity of the drums upon impact with the ocean floor. To this end, we digress briefly to study elementary Newtonian mechanics.

Newtonian mechanics is the study of Newton's famous laws of motion and their consequences. Newton's first law of motion states that an object will remain at rest, or move with constant velocity, if no force is acting on it. A force should be thought of as a push or pull. This push or pull can be exerted directly by something in contact with the object, or it can be exerted indirectly, as the earth's pull of gravity is.

Newton's second law of motion is concerned with describing the motion of an object, which is acted upon by several forces. Let y(t) denote the position of the center of gravity of the object. (We assume that the object moves. in only one direction.).

Those forces acting on the object, which tend to increase y, are considered positive, while those forces tending to decrease y are considered negative. The resultant force F acting on an object is defined to be the sum of all positive forces minus the sum of all negative forces. Newton's second law of motion states that the acceleration d^2y/dt^2 of an object is proportional to the resultant force F acting on it; i.e.,

$$d^2y/dt^2 = F/m \qquad (1)$$

The constant m is the mass of the object. It is related to the weight W of the object by the relation W = mg, where g is the acceleration of gravity. Unless otherwise stated, we assume that the weight of an object and the acceleration of gravity are constant. We will also adopt the English system of units, so that t is measured in seconds, y is measured in feet, and F is measured in pounds. The units of m are then slugs, and the gravitational acceleration g equals 32.2 ft/s^2.

H-function of one variable which is introduced by Fox [1, p.408], will be represented as follows:

$$H^{m,n}_{p,q}\left[x \,\middle|\, {(a_j, \alpha_j)_{1,p} \atop (b_j, \beta_j)_{1,q}}\right] = (1/2\pi i) \int_L \theta(s)\, x^s\, ds \qquad (A)$$

where $i = \sqrt{(-1)}$,

$$\theta(s) = \frac{\prod_{j=1}^{m} \Gamma(b_j - \beta_j s) \prod_{j=1}^{n} \Gamma(1 - a_j + \alpha_j s)}{\prod_{j=m+1}^{q} \Gamma(1 - b_j + \beta_j s) \prod_{j=n+1}^{p} \Gamma(a_j - \alpha_j s)}$$

x is not equal to zero and an empty product is interpreted as unity; p, q, m, n are integers satisfying $1 \le m \le q$, $0 \le n \le p$, α_j (j = 1, …., p), β_j (j = 1, …, q) are positive numbers and a_j (j = 1, …, q) are complex numbers. L is a suitable contour of Barnes type such that poles of $\Gamma(b_j - \beta_j s)$ (j = 1, …, m) lie to the right and poles of $\Gamma(1 - a_j + \alpha_j s)$ (j = 1, …, n) to the left of L. These assumptions for the H-function will be adhered to through out this paper.

According to Braakasma

$$H^{m,n}_{p,q}\left[x \,\middle|\, {(a_j, \alpha_j)_{1,p} \atop (b_j, \beta_j)_{1,q}}\right] = O(|x|^{\alpha}) \text{ for small } x,$$

where $\sum_{j=1}^{p} \alpha_j - \sum_{j=1}^{q} \beta_j \le 0$ and $\alpha = \min R(b_h/\beta_h)$ (h = 1, .., k) and

$$H^{m,n}_{p,q}\left[x\left|\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right.\right] = O\left(|x|^{\beta}\right) \text{ for large } x,$$

where

$$\sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p}\alpha_j + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q}\beta_j \equiv A > 0,$$

$$\sum_{j=1}^{p}\alpha_j - \sum_{j=1}^{q}\beta_j < 0$$

$|\arg x| < \frac{1}{2} A\pi$ and $\beta = \max R[(a_j - 1)/\alpha_j]$ $(j = 1, .., n)$

2. MATHEMATICAL MODEL:

We return now to our atomic waste disposal problem. As a drum descends through the water, it is acted upon by three forces W, B, and D. The force W is the weight of the drum pulling it down, and in magnitude, W = 527.436 lb. The force B is the buoyancy force of the water acting on the drum. This force pushes the drum up, and its magnitude is the weight of the water displaced by the drum. Now, the Atomic Energy Commission used 55 gallon drums, whose volume is 7.35 ft^3. The weight of one cubic foot of salt water is 63.99lb. Hence B = (63.99) (7.35) = 470.327 lb.

The force D is the drag force of the water acting on the drum; it resists the motion of the drum through the water. Experiments have shown that any medium such as water, oil, and air resists the motion of an object through it. This resisting force acts in the direction opposite the motion, and is usually directly proportional to the velocity V of the object. Thus, D = cV, for some positive constant c. Notice that the drag force increases, as V increases, and decreases as V decreases. To calculate D, the engineers conducted numerous towing experiments. They concluded that the orientation of the drum had little effect on the drag force, and that D = 0.08 V (lb)(s)/ft.

Now, set y = 0 at sea level, and let the direction of increasing y be downwards. Then, W is a positive force, and B and D are negative forces. Consequently, from (1),

$$d^2y/dt^2 = (W - B - cV)/m = (g/W)(W - B - cV).$$

We can rewrite this equation as a first-order linear differential equation for $V = dy/dt$;

i.e. $$dV/dt + (cg/W)V = (g/W)(W - B). \quad (2)$$

Initially, when the drum is released in the ocean, its velocity is zero. Thus, V (t), the velocity of the drum, satisfies the initial-value problem

$$dV/dt + (cg/W)V = (g/W)(W - B), \quad V(0) = 0. \qquad (3)$$

and this implies that;

$$V(t) = [(W - B)/c]\,[1 - e^{(-c\,g/W)\,t}]. \qquad (4)$$

Equation (4) expresses the velocity of the drum as a function of time. In order to determine the impact velocity of the drum, we must compute the time t at which the drum hits the ocean floor. Unfortunately, though, it is impossible to find t as an explicit function of y. Therefore, we cannot use Equation (4) to find the velocity of the drum when it hits the ocean floor. However, the A.E.C. can use this equation to try and prove that the drums do not crack on impact. To overcome this problem here we are giving a solution of equation (3) in terms of H-function, which can be helpful to determine the solution of the above raised problem, since H-function may be reduced to Legendre functions, Bessel functions etc.

3. SOLUTION IN TERMS OF H-FUNCTION:

Choose concentration V(t) in terms of H-function as

$$V(t) = H^{m,\,n}_{p,\,}\left[z\,t^{\mu}\,\middle|\,\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_i,\beta_i)_{1,q}\end{matrix}\right] \qquad (5)$$

where $\mu > 0$, $|\arg z| < \frac{1}{2}\pi A$, where A is given in Section 1.

Now differentiate it with respect to t, we get

$$dV/dt = (1/t)\,H^{m,\,n+1}_{p+1,\,q+1}\left[z\;t^{\mu}\,\middle|\,\begin{matrix}(0,\mu),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},(1,\mu)\end{matrix}\right] \qquad (6)$$

Now after using (5) and (6) in (3), we get following result

$$(1/t)\,H^{m,\,n+1}_{p+1,\,q+1}\left[z\;t^{\mu}\,\middle|\,\begin{matrix}(0,\mu),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},(1,\mu)\end{matrix}\right]$$

$$+ (Cg/W)\,H^{m,\,n}_{p,\,}\left[z\,t^{\mu}\,\middle|\,\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_i,\beta_i)_{1,q}\end{matrix}\right] = (g/W)(W - B), \quad V(0) = 0. \qquad (7)$$

where $\mu > 0$, $|\arg z| < \frac{1}{2}\pi A$.

4. SPECIAL CASES:

On specializing the parameters, H-function may be reduced to G-function, Lauricella's functions Legendre functions, Bessel functions, hypergeometric functions, Appell's functions, Kampe de Feriet's functions and several other higher transcendental functions. Therefore the result (7) is of general nature and may reduced to be in different forms, which will be useful in the literature on applied Mathematics and other branches.

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NERVE EXCITATION AND FOX'S H-FUNCTION

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ABSTRACT

The aim this paper is to study the Nerve Excitation involving Fox's H-Function of one variable.

1. INTRODUCTION:

The H-function of one variable which is introduced by Fox [1, p.408], will be represented as follows:

$$H_{p,q}^{m,n}\left[x \middle| \begin{matrix}(a_j, \alpha_j)_{1,p}\\(b_j, \beta_j)_{1,q}\end{matrix}\right] = (1/2\pi i)\int_L \theta(s)\, x^s\, ds \tag{A}$$

where $i = \sqrt{(-1)}$,

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\ \prod_{j=1}^{n}\Gamma(1 - a_j + \alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1 - b_j + \beta_j s)\ \prod_{j=n+1}^{p}\Gamma(a_j - \alpha_j s)}$$

x is not equal to zero and an empty product is interpreted as unity; p, q, m, n are integers satisfying $1 \le m \le q$, $0 \le n \le p$, α_j (j = 1, …., p), β_j (j = 1, …, q) are positive numbers and a_j (j = 1, …, q) are complex numbers. L is a suitable contour of Barnes type such that poles of $\Gamma(b_j - \beta_j s)$ (j = 1, …, m) lie to the right and poles of $\Gamma(1 - a_j + \alpha_j s)$ (j = 1, …, n) to the left of L. These assumptions for the H-function will be adhered to through out this paper.

According to Braakasma

$$H_{p,q}^{m,n}\left[x \middle| \begin{matrix}(a_j, \alpha_j)_{1,p}\\(b_j, \beta_j)_{1,q}\end{matrix}\right] = O(|x|^{\alpha}) \text{ for small } x,$$

where $\sum_{j=1}^{p}\alpha_j - \sum_{j=1}^{q}\beta_j \le 0$ and $\alpha = \min R(b_h/\beta_h)$ (h = 1, .., k)

and

$$H_{p,q}^{m,n}\left[x \middle| \begin{matrix}(a_j, \alpha_j)_{1,p}\\(b_j, \beta_j)_{1,q}\end{matrix}\right] = O(|x|^{\beta}) \text{ for large } x,$$

where

$$\sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p}\alpha_j + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q}\beta_j \equiv A > 0,$$

$$\sum_{j=1}^{p} \alpha_j - \sum_{j=1}^{q} \beta_j < 0$$

$|\arg x| < ½ A\pi$ and $\beta = \max R[(a_j - 1)/\alpha_j]$ (j = 1, .., n)

The cells of a nerve fibre may be conceived as an electric system. The protoplasm contains a large number of different ions, both cations (positive electric charge) and anions (negative electric charge). When an electric current is applied to a nerve fibre, the cations move to the cathode, the anions to the anode, and the electric equilibrium are disturbed. This phenomenon leads to the excitation of the nerve.

Based on the observation that the excitation originates at the cathode, N. Rashevsky, developed a theory which postulates that two different kinds of cations are responsible for the process. One is exciting and the other kind is inhibiting. These two kinds are said to be antagonistic factors.

2. MATHEMATICAL MODEL:

Let E = E(t) be the concentration of the exciting cations and F = F(t) be the concentration of the inhibiting cations near the cathode at any time t. The theory then states that excitation occurs whenever the ratio E/F exceeds a certain value. Denoting this value by c, we have excitation when $E/F \geq C$ and there will be no excitation if $E/F < C$. Let E_0 and F_0 be the concentrations at rest of exciting and inhibiting cations, respectively. When E increases and F remains limited, there is excitation. When E does not increase as fast as F, then there is no excitation.

Let I be the intensity of the stimulant current. For convenience sake, assume that I is constant during a certain time interval. Rashevsky showed that the Excitation of nerves can be described by the differential equations

$$dE/dt = JI - K(E - E_0) \quad (1)$$

and

$$dF/dt = LI - M(F - F_0) \quad (2)$$

where J, K, L, M are positive constants.

The above equations can be easily solved for E and F, and finally the ratio E/F determines whether excitation occurs and when.

3. SOLUTION IN TERMS OF H-FUNCTION:

Choose E(t) and F(t) concentration of the exciting cations and concentration of the inhibiting cations respectively in terms of H-function (A) as

$$E(t) = H^{m,n}_{p,q}\left[z t^{\lambda} \,\middle|\, {(a_j, \alpha_j)_{1,p} \atop (b_i, \beta_i)_{1,q}}\right] \tag{3}$$

$$F(t) = H^{m,n}_{p,q}\left[z t^{\mu} \,\middle|\, {(a_j, \alpha_j)_{1,p} \atop (b_i, \beta_i)_{1,q}}\right] \tag{4}$$

where $\lambda > 0$, $\mu > 0$, $|\arg z| < \frac{1}{2}\pi A$, where A is given in section 1.

Now differentiate (3) and (4) with respect to t, we get

$$dE/dt = (1/t)\, H^{m,n+1}_{p+1,q+1}\left[z\ t^{\lambda} \,\middle|\, {(0, \lambda), (a_j, \alpha_j)_{1,p} \atop (b_j, \beta_j)_{1,q}, (1, \lambda)}\right] \tag{5}$$

and

$$dF/dt = (1/t)\, H^{m,n+1}_{p+1,q+1}\left[z\ t^{\mu} \,\middle|\, {(0, \mu), (a_j, \alpha_j)_{1,p} \atop (b_j, \beta_j)_{1,q}, (1, \mu)}\right] \tag{6}$$

Now after using (3), (4), (5) and (6) in (1) and (2), we get following result

$$(1/t)\, H^{m,n+1}_{p+1,q+1}\left[z\ t^{\lambda} \,\middle|\, {(0, \lambda), (a_j, \alpha_j)_{1,p} \atop (b_j, \beta_j)_{1,q}, (1, \lambda)}\right]$$

$$= JI - K\left(H^{m,n}_{p,}\left[z t^{\lambda} \,\middle|\, {(a_j, \alpha_j)_{1,p} \atop (b_i, \beta_i)_{1,q}}\right] - E_0\right) \tag{7}$$

$$(1/t)\, H^{m,n+1}_{p+1,q+1}\left[z\ t^{\mu} \,\middle|\, {(0, \mu), (a_j, \alpha_j)_{1,p} \atop (b_j, \beta_j)_{1,q}, (1, \mu)}\right]$$

$$= LI - M\left(H^{m,n}_{p,}\left[z t^{\lambda} \,\middle|\, {(a_j, \alpha_j)_{1,p} \atop (b_i, \beta_i)_{1,q}}\right] - F_0\right) \tag{8}$$

where $\lambda > 0$, $\mu > 0$, $|\arg z| < \frac{1}{2}\pi A$, where A is given in section 1.

4. SPECIAL CASES:

On specializing the parameters, H-function may be reduced to G-function, Lauricella's functions Legendre functions, Bessel functions, hypergeometric functions, Appell's functions, Kampe de Feriet's functions and several other higher transcendental functions. Therefore the result (7) and (8) is of general nature and may reduced to be in different forms, which will be useful in the literature on applied Mathematics and other branches.

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FIXED POINTS OF FUZZY MAPPINGS IN HILBERT SPACES

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ABSTRACT: In this paper we proved two fixed point theorems for fuzzy mappings in Hilbert spaces by using Parallelogram law. Our result includes several fixed point results in ordinary metric spaces as special cases.

KEYWORDS: Hilbert spaces, fixed point, fuzzy mapping, approximate quantity.

MATHEMATICS SUBJECT CLASSIFICATION: Primary 47H10, Secondary 54H25

1. INTRODUCTION:

Heilpern [7] introduced the concept of fuzzy mappings as a mapping from an arbitrary set to one subfamily of fuzzy sets in a metric linear space and proved a fixed point theorem for fuzzy mappings. Many authors extended and generalized Heilpern's result [6], [1], [2], [8] and [9]. In the present paper, we prove some fixed point theorems of fuzzy mappings which generalize the result of Heilpern [7].

To establish our main result we need the following definitions:

2. PRELIMINARIES:

Definition 2.1([7]): Let H be a Hilbert space and F(H) be a collection of all fuzzy sets in H. Let $A \in F(H)$ and $\alpha \in [0,1]$, then the α-level set of A, is denoted by A_α is defined as

$$A_\alpha = \{x : A(x) \geq \alpha, \text{ if } \alpha \in (0, 1]\}$$
$$A_0 = \{\overline{x} : A(x) > 0\}$$

where $\overline{B}$ stads for the closure of a set B.

Definition 2.2([7]) : A fuzzy subset A of H is said to be an approximate quantity if and only if its α-level set is non-fuzzy compact convex subset of H for each $\alpha \in (0, 1]$ and $\sup_{x \in H} A(x) = 1$.

From the collection F(H), the sub collection of all approximate quantities is denoted by W(H).

Definition 2.3([7]) : Let $A, B \in W(H)$ and $\alpha \in [0, 1]$, then

(1) $P_\alpha(A, B) = \inf_{x \in A\alpha,\, y \in B\alpha} \|x - y\|$,
(2) $D_\alpha(A, B) = \text{dist}(A_\alpha, B_\alpha)$ where 'dist' denotes the Housdorff metric between A_α and B_α,
(3) $D(A, B) = \sup_\alpha D_\alpha(A, B)$ and
(4) $P(A, B) = \sup_\alpha P_\alpha(A, B)$.

It is to be noted that for any 'α' P_α is a non-decreasing as well as continuous function.

Definition 2.4([7]) : Let A, B ∈ W(H). An approximate quantity A is said to be more accurate then B, denoted by A ⊂ B, iff A(x) < B(x) for each x ∈ H. The relation ⊂ induced a parallel ordering on W(H).

Definition 2.5([7]) : A mapping F from the set H onto W(H) is said to be a fuzzy mapping. Any x ∈ H is called a fixed point of the mapping H : H → W(H) if {x} ⊂ Fx, where {x} is the fuzzy set with membership function equal to the characteristic function of crisp set {x}.

We shall use the following lemmas due to Heilpern [ ].

Lemma 2.6([7]) : Let x ∈ H and A ∈ W(H), then {x} ⊂ A if and only if P_α (x, A) = 0 for each α ∈ [0, 1]

Lemma 2.7([7]) : $P_\alpha(x, A) \le \|x-y\| + P_\alpha(y, A)$, for each x, y ∈ H.

Lemma 2.8([7]) : If $\{x_0\} \subset A$, then $P_\alpha(x_0, A) \le D(A, B)$ for each B ∈ W(H).

3. MAIN RESULTS:

Theorem 3.1 : Let H be a Hilbert space, F and G are fuzzy mappings from H into W(H) satisfying

(3.1.1) $$D^2(Fx, Gy) \le a_1\|x-y\|^2 + a_2P_\propto^2(x,Fx) + a_3P_\propto^2(y,Gy) + a_4P_\propto^2(y,Fx) + a_5P_\propto^2(x,Gy) + a_6\left\{\frac{P_\propto^2(y,Gy)\,[1+P_\propto^2(x,Fx)]}{1+\|x-y\|^2}\right\}$$

For all x, y ∈ H and for all α ∈ [0, 1] and a_1, a_2, a_3, a_4, a_5 and a_6 are non-negative numbers satisfying

(3.1.2) $a_1 + a_2 + a_3 + 4a_5 + a_6 < 1$

Then Fz and Gz have a common fixed point in H.

Proof : Let x_0 ∈ H. We construct a sequence $\{x_n\}$ as follows:

$\{x_1\} \subset Fx_0$, $\{x_2\} \subset Gx_0$, . . . $\{x_{2n+1}\} \subset Fx_{2n}$, $\{x_{2n+2}\} \subset Gx_{2n+1,}$

and $\|x_i - x_{i+1}\| \le D(Fx_{i-1,} Gx_{i,})$, i = 1, 2, 3, . . .

From (3.3.1), we have

$$\|x_{2n} - x_{2n+1}\|^2 = D^2(Fx_{2n-1}, Gx_{2n})$$
$$\le a_1\|x_{2n-1}-x_{2n}\|^2 + a_2P_\propto^2(x_{2n-1},Fx_{2n-1}) + a_3P_\propto^2(x_{2n},Gx_{2n}) + a_4P_\propto^2(x_{2n},Fx_{2n-1}) + a_5P_\propto^2(x_{2n-1},Gx_{2n}) + a_6\left\{\frac{P_\propto^2(x_{2n},Gx_{2n})\,[1+P_\propto^2(x_{2n-1},Fx_{2n-1})]}{1+\|x_{2n-1}-x_{2n}\|^2}\right\}$$
$$= a_1\|x_{2n-1}-x_{2n}\|^2 + a_2\|x_{2n-1}-x_{2n}\|^2 + a_3\|x_{2n}-x_{2n+1}\|^2 + a_4\|x_{2n}-x_{2n}\|^2 + a_5\|x_{2n-1}-x_{2n+1}\|^2 + a_6\left\{\frac{\|x_{2n}-x_{2n+1}\|^2\,[1+\|x_{2n-1}-x_{2n}\|^2]}{1+\|x_{2n-1}-x_{2n}\|^2}\right\}$$
$$= a_1\|x_{2n-1}-x_{2n}\|^2 + a_2\|x_{2n-1}-x_{2n}\|^2 + a_3\|x_{2n}-x_{2n+1}\|^2 + a_5\{2\|x_{2n-1}-x_{2n}\|^2 + 2\|x_{2n}-x_{2n+1}\|^2\} + a_6\|x_{2n}-x_{2n+1}\|^2$$

By using parallelogram law $\|x+y\|^2 \le 2\|x\|^2 + 2\|y\|^2$

which gives

(3.1.3) $\|x_{2n} - x_{2n+1}\|^2 \leq \lambda \|x_{2n-1} - x_{2n}\|^2$
where

$$0 < \lambda = \frac{a_1 + a_2 + 2a_5}{1-(a_3 + 2a_5 + a_6)} < 1$$

Again,

$$\|x_{2n-1} - x_{2n}\|^2 = D^2(Fx_{2n-2}, Gx_{2n-1})$$
$$\leq a_1\|x_{2n-2} - x_{2n-1}\|^2 + a_2 P_\propto^2(x_{2n-2}, Fx_{2n-2}) + a_3 P_\propto^2(x_{2n-1}, Gx_{2n-1}) + a_4 P_\propto^2(x_{2n-1}, Fx_{2n-2}) + a_5 P_\propto^2(x_{2n-2}, Gx_{2n-1}) + a_6\left\{\frac{P_\propto^2(x_{2n-1}, Gx_{2n-1})\,[1+ P_\propto^2(x_{2n-2}, Fx_{2n-2})]}{1+ \|x_{2n-2} - x_{2n-1}\|^2}\right\}$$
$$= a_1\|x_{2n-2} - x_{2n-1}\|^2 + a_2\|x_{2n-2} - x_{2n-1}\|^2 + a_3\|x_{2n-1} - x_{2n}\|^2 + a_4\|x_{2n-1} - x_{2n-1}\|^2 + a_5\|x_{2n-2} - x_{2n}\|^2 + a_6\left\{\frac{\|x_{2n-1} - x_{2n}\|^2\,[1+ \|x_{2n-2} - x_{2n-1}\|^2]}{1+ \|x_{2n-2} - x_{2n-1}\|^2}\right\}$$
$$= a_1\|x_{2n-2} - x_{2n-1}\|^2 + a_2\|x_{2n-2} - x_{2n-1}\|^2 + a_3\|x_{2n-1} - x_{2n}\|^2 + a_5\{2\|x_{2n-2} - x_{2n-1}\|^2 + 2\|x_{2n-1} - x_{2n}\|^2\} + a_6\|x_{2n-1} - x_{2n}\|^2$$

By using parallelogram law $\|x + y\|^2 \leq 2\|x\|^2 + 2\|y\|^2$

which gives

(3.1.4) $\|x_{2n-1} - x_{2n}\|^2 \leq \lambda \|x_{2n-2} - x_{2n-1}\|^2$
where

$$0 < \lambda = \frac{a_1 + a_2 + 2a_5}{1-(a_3 + 2a_5 + a_6)} < 1$$

In general, it follows that

$$\|x_{n+1} - x_n\|^2 \leq \lambda \|x_n - x_{n-1}\|^2 \quad (0 < \lambda < 1)$$

Hence, $\{x_n\}$ be a Cauchy sequence in H and therefore it converges to a limit in H.

Let, $\lim_{n\to\infty} x_n = z$.

Again using lemma(2.7) and for all $\alpha \in [0, 1]$, we have from (3.1.1)

$$P_\propto^2(x_{2n+2}, Fz) \leq D_\propto^2(Gx_{2n+1}, Fz)$$
$$\leq a_1\|z - x_{2n+1}\|^2 + a_2 P_\propto^2(z, Fz) + a_3 P_\propto^2(x_{2n+1}, Gx_{2n+1}) + a_4 P_\propto^2(x_{2n+1}, Fz) + a_5 P_\propto^2(z, Gx_{2n+1}) + a_6\left\{\frac{P_\propto^2(x_{2n+1}, Gx_{2n+1})\,[1+ P_\propto^2(z, Fz)]}{1+ \|z - x_{2n+1}\|^2}\right\}$$
$$= a_1\|z - x_{2n+1}\|^2 + a_2 P_\propto^2(z, Fz) + a_3 P_\propto^2(x_{2n+1}, x_{2n+2}) + a_4 P_\propto^2(x_{2n+1}, Fz) + a_5 P_\propto^2(z, x_{2n+2}) + a_6\left\{\frac{P_\propto^2(x_{2n+1}, x_{2n+2})\,[1+ P_\propto^2(z, Fz)]}{1+ \|z - x_{2n+1}\|^2}\right\}$$

Making $n \to \infty$ and using the fact that P_α is continuous,

$$P_\propto^2(x_{2n+2}, Fz) \leq (a_2 + a_4)\, P_\propto^2(z, Fz)$$

Since, $a_2 + a_4 < 1$, It follows that $P_\propto^2(z, Fz) = 0$.

Therefore, by lemma (2.5), $\{z\} \subset Fz$.

Similarly, {z} ⊂ Gz.
Hence, {z} ⊂ Fz ∩ Gz.
Thus z is a common fixed point of F and G.
This completes the proof of the theorem.

Theorem 3.2: Let H be a Hilbert space, F and G are fuzzy mappings from H into W(H) satisfying

(3.2.1) $D^2(Fx, Gy) \leq q \max\{\|x-y\|^2, P_\propto^2(x,Fx), P_\propto^2(y,Gy), P_\propto^2(y,Fx), P_\propto^2(x,Gy), \frac{P_\propto^2(y,Gy)\,[1+P_\propto^2(x,Fx)]}{1+\|x-y\|^2}\}$

For all x, y ∈ H and for all α ∈ [0, 1] and $0 < q < \frac{1}{4}$

Then Fz and Gz have a common fixed point.

Proof: Let $x_0 \in H$. We construct a sequence $\{x_n\}$ as in theorem (3.1) and correspondingly,
we have

$$\|x_{2n} - x_{2n+1}\|^2 = D^2(Fx_{2n-1}, Gx_{2n})$$
$$\leq q \max\{\|x_{2n-1} - x_{2n}\|^2, P_\propto^2(x_{2n-1},Fx_{2n-1}), P_\propto^2(x_{2n},Gx_{2n}), P_\propto^2(x_{2n},Fx_{2n-1}), P_\propto^2(x_{2n-1},Gx_{2n}), \frac{P_\propto^2(x_{2n},Gx_{2n})\,[1+P_\propto^2(x_{2n-1},Fx_{2n-1})]}{1+\|x_{2n-1}-x_{2n}\|^2}\}$$

$$= q \max\{\|x_{2n-1} - x_{2n}\|^2, P_\propto^2(x_{2n-1}, x_{2n}), P_\propto^2(x_{2n},x_{2n+1}), P_\propto^2(x_{2n},x_{2n}), P_\propto^2(x_{2n-1},x_{2n+1}), \frac{P_\propto^2(x_{2n},x_{2n+1})\,[1+P_\propto^2(x_{2n-1},x_{2n})]}{1+\|x_{2n-1}-x_{2n}\|^2}\}$$

$$= q \max\{\|x_{2n-1} - x_{2n}\|^2, \|x_{2n-1} - x_{2n}\|^2, \|x_{2n} - x_{2n+1}\|^2, \|x_{2n} - x_{2n}\|^2, \|x_{2n-1} - x_{2n+1}\|^2, \frac{\|x_{2n}-x_{2n+1}\|^2\,[1+\|x_{2n-1}-x_{2n}\|^2]}{1+\|x_{2n-1}-x_{2n}\|^2}\}$$

$$= q \max\{\|x_{2n-1} - x_{2n}\|^2, \|x_{2n} - x_{2n+1}\|^2, 2\{\|x_{2n-1} - x_{2n}\|^2 + \|x_{2n} - x_{2n+1}\|^2\}$$

By using parallelogram law $\|x+y\|^2 \leq 2\|x\|^2 + 2\|y\|^2$

Therefore,

$$\|x_{2n} - x_{2n+1}\|^2 \leq 2q\,\{\|x_{2n-1} - x_{2n}\|^2 + \|x_{2n} - x_{2n+1}\|^2\}$$

which yields,

(3.2.2) $\|x_{2n} - x_{2n+1}\|^2 \leq \frac{2q}{1-2q}\|x_{2n-1} - x_{2n}\|^2$

Again,

$$\|x_{2n-1} - x_{2n}\|^2 = D^2(Fx_{2n-2}, Gx_{2n-1})$$
$$\leq q\max\{\|x_{2n-2} - x_{2n-1}\|^2, P_\propto^2(x_{2n-2},Fx_{2n-2}), P_\propto^2(x_{2n-1},Gx_{2n-1}), P_\propto^2(x_{2n-1},Fx_{2n}), P_\propto^2(x_{2n-2},Gx_{2n-1}), \frac{P_\propto^2(x_{2n-1},Gx_{2n-1})\,[1+P_\propto^2(x_{2n-2},Fx_{2n-2})]}{1+\|x_{2n-2}-x_{2n-1}\|^2}\}$$
$$= q\max\{\|x_{2n-2} - x_{2n-1}\|^2, P_\propto^2(x_{2n-2}, x_{2n-1}), P_\propto^2(x_{2n-1},x_{2n}), P_\propto^2(x_{2n-1},x_{2n-1}), P_\propto^2(x_{2n-2},x_{2n}),$$

$\frac{P_{\propto}^2(x_{2n-1},x_{2n})\,[1+P_{\propto}^2(x_{2n-2},x_{2n-1})]}{1+\|x_{2n-2}-x_{2n-1}\|^2}\}$

$= \text{q}\max\{\|x_{2n-2}-x_{2n-1}\|^2,\ \|x_{2n-2}-x_{2n-1}\|^2, \|x_{2n-1}-x_{2n}\|^2,$

$\|x_{2n-1}-x_{2n-1}\|^2, \|x_{2n-2}-x_{2n}\|^2, \frac{\|x_{2n-1}-x_{2n}\|^2\,[1+\|x_{2n-2}-x_{2n-1}\|^2]}{1+\|x_{2n-2}-x_{2n-1}\|^2}\}$

$= \text{q}\max\{\|x_{2n-2}-x_{2n-1}\|^2, \|x_{2n-1}-x_{2n}\|^2,$

$2\{\|x_{2n-2}-x_{2n-1}\|^2 + \|x_{2n-1}-x_{2n}\|^2\}$

By using parallelogram law $\|x+y\|^2 \le 2\|x\|^2 + 2\|y\|^2$

Therefore,

$\|x_{2n-1}-x_{2n}\|^2 \le 2\text{q}\,\{\|x_{2n-2}-x_{2n-1}\|^2 + \|x_{2n-1}-x_{2n}\|^2\}$

which yields,

(3.2.3) $\|x_{2n-1}-x_{2n}\|^2 \le \frac{2q}{1-2q}\|x_{2n-2}-x_{2n-1}\|^2$

Therefore, from (3.2.2) and (3.2.3) , it follows that

$\|x_{n+1}-x_n\|^2 \le \lambda\,\|x_n-x_{n-1}\|^2 \quad (0 < \lambda < 1)$

Hence, $\{x_n\}$ be a Cauchy sequence in H and therefore it converges to a limit in H. Let, $\lim_{n\to\infty} x_n = z$.

Again using lemma(2.7) and for all α ∈ [0, 1], we have from (3.2.1)

$P_{\propto}^2(x_{2n+2}, \text{Fz}) \le D_{\propto}^2(Gx_{2n+1}, \text{Fz})$

$\le \text{q}\max\{\|z-x_{2n+1}\|^2, P_{\propto}^2(z,Fz),\ P_{\propto}^2(x_{2n+1},Gx_{2n+1}),$

$P_{\propto}^2(x_{2n+1},Fz), P_{\propto}^2(z,Gx_{2n+1}), \frac{P_{\propto}^2(x_{2n+1},Gx_{2n+1})\,[1+P_{\propto}^2(z,Fz)]}{1+\|z-x_{2n+1}\|^2}\}$

$\le \text{q}\max\{\|z-x_{2n+1}\|^2, P_{\propto}^2(z,Fz),\ P_{\propto}^2(x_{2n+1},x_{2n+2}),$

$P_{\propto}^2(x_{2n+1},Fz), P_{\propto}^2(z,x_{2n+2}), \frac{P_{\propto}^2(x_{2n+1},x_{2n+2})\,[1+P_{\propto}^2(z,Fz)]}{1+\|z-x_{2n+1}\|^2}\}$

Making n → ∞ and using the fact that P_α is continuous,

$P_{\propto}^2(x_{2n+2}, \text{Fz}) \le \text{q}P_{\propto}^2(z,Fz)$

Since, $0 < q < \frac{1}{4}$, It follows that $P_{\propto}^2(z,Fz) = 0$.

Therefore, by lemma (2.5), {z} ⊂ Fz.

Similarly, {z} ⊂ Gz.

Hence, {z} ⊂ Fz ∩ Gz.

Thus z is a common fixed point of F and G.

This completes the proof of the theorem.

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